

2501/203 2508/203

2502/203 2509/203

2503/203

**ENGINEERING MATHEMATICS II**

Oct./Nov. 2016

Time: 3 hours



**THE KENYA NATIONAL EXAMINATIONS COUNCIL**  
**DIPLOMA IN MECHANICAL ENGINEERING**  
**(PRODUCTION OPTION)**  
**(PLANT OPTION)**  
**DIPLOMA IN AUTOMOTIVE ENGINEERING**  
**DIPLOMA IN WELDING AND FABRICATION**  
**DIPLOMA IN CONSTRUCTION PLANT ENGINEERING**

**MODULE II**

**ENGINEERING MATHEMATICS II**

**3 hours**

**INSTRUCTIONS TO CANDIDATES**

*You should have the following for this examination:*

*Answer booklet;*

*Mathematical tables/ Non-programmable scientific calculator;*

*Drawing instruments.*

*This paper consists of EIGHT questions.*

*Answer any FIVE questions in the answer booklet provided.*

*All questions carry equal marks.*

*Maximum marks for each part of a question are as shown.*

*Candidates should answer the questions in English.*

**This paper consists of 5 printed pages.**

**Candidates should check the question paper to ascertain that  
all the pages are printed as indicated and that no questions are missing.**

© 2016 The Kenya National Examinations Council

Turn over

1. (a) Evaluate  $\int_0^{\pi/2} 3x^2 \cos x \, dx$  giving your answer to three decimal places. (5 marks)
- (b) Determine  $\int \frac{9x^2 - 73x + 150}{(x-7)(x-3)^2} \, dx$  (6 marks)
- (c) (i) Find the area bounded by the curve given by the parametric equation  $x = 2t^2$ ,  $y = 2t - t^2$  and the x axis, between  $t = 0$  and  $t = 2$ . (4 marks)
- (ii) Determine the volume generated when the area in (i) above revolves about the x axis through  $360^\circ$ . (5 marks)
2. (a) Given that  $u = \frac{x+y+z}{(x^2+y^2+z^2)^{3/2}}$ , prove that  $x \frac{du}{dx} + y \frac{du}{dy} + z \frac{du}{dz} = 0$ . (7 marks)
- (b) The deflection  $y$  at the centre of a uniformly loaded plate suspended at the edge is given by  $y = \frac{kwd^4}{t^3}$ , where  $w$  is the total load,  $d$  the diameter of the plate,  $t$  the thickness of plate and  $k$  is a constant. Use partial differentiation to calculate the approximate percentage change in  $y$  if  $w$  is increased by 2%,  $d$  is decreased by 1% and  $t$  increased by 3%. (5 marks)
- (c) A rectangular tank which is open at the top holds 108 cubic metre of water. Determine its dimension so that the total surface area is a minimum. (8 marks)
3. (a) Use Maclaurin's series to expand  $e^{\sin x}$  in ascending powers of  $x$  up to and including the term in  $x^4$ . Hence evaluate  $\int_0^1 \frac{e^{\sin x}}{x^2} \, dx$  giving your answer correct to four decimal places. (12 marks)
- (b) Use Taylor's theorem to expand  $\cos(a+h)$  in ascending powers of  $h$  as far as the term in  $h^3$ . Hence determine  $\cos 31^\circ$  correct to five decimal places. (8 marks)
4. (a) Given that  $y = \ln \left( \frac{\cos x + \sin x}{\cos x - \sin x} \right)$ , show that  $\frac{dy}{dx} = 2 \operatorname{Sec} 2x$ . (7 marks)
- (b) If  $x^2y + xy^2 - x^3 + y^3 + 16 = 0$ , find, using implicit differentiation  $\frac{dy}{dx}$  at the point  $(1, 3)$ . (4 marks)

- (c) The motion of a body undergoing damped vibration is given by  $y = e^{-t} \sin 2t$ ,  $y$  being the displacement from its mean position at time  $t$ .
- (i) Show that  $y$  is maximum when  $t = \frac{1}{2} \tan^{-1} 2$ .
- (ii) Hence determine the maximum displacement to 3 decimal places. (9 marks)

5. (a) Table 1 shows the distribution of marks scored by 230 students in a mathematics test. If the median and mode marks are 33.5 and 34 respectively, determine the values of  $x$ ,  $y$  and  $z$ . (12 marks)

Table 1

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of students	4	16	$x$	$y$	$z$	6	4

- (b) Table 2 shows the frequency distribution of lengths of 50 copper plugs. Calculate the mean and standard deviation of the distribution.

Table 2

Length in (mm)	14.0-14.2	14.3-14.5	14.6-14.8	14.9-15.1	15.2-15.4	15.5-15.7	15.8-16.0
Frequency	2	4	9	15	11	6	3

(8 marks)

6. (a) Evaluate the integral  $\int_0^{\pi/8} \sqrt{\sin 2x} \, dx$  using Simpson's rule with six intervals, correct to four decimal places. (8 marks)
- (b) A pyramid has a rectangular base 3.40 cm by 5.20 cm. If each of the sloping edges is 14.80 cm, determine the volume and total surface area of the pyramid. (12 marks)
7. (a) The first term of both arithmetic progression and geometric progression is 2. The common ratio is equal to common difference. If the third term of the geometric progression exceeds the square of the first term of an arithmetic progression by 124, determine the:
- (i) common difference;
- (ii) sum of the first 10 terms of the arithmetic progression. (7 marks)

- (b) Two cars P and Q, are travelling towards the junction of two roads which are at right angles to one another. Car P has a velocity of 40 km/h due east and car Q a velocity of 50 km/h due south. Calculate the:
- velocity of car P relative to car Q;
  - velocity of car Q relative to car P.

(6 marks)

- (c) A packet contains 100 washers, 24 of which are brass, 36 copper and the remainder steel. One washer is taken, tested but not replaced, a second washer is treated similarly. Determine the probability that:

- both washers are steel;
- the first is brass and second copper;
- one brass and one is steel.

(7 marks)

8. (a) A continuous random variable  $t$  has a probability density function defined by

$$f(t) = \begin{cases} \frac{k^2}{2} e^{-kt} & t \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

where  $k$  is a constant.

$\frac{e}{2k}$

Determine the:

- value of  $k$ ;
- mean;
- median.

(12 marks)

- (b) The diameter of metal bolts produced by a factory is normally distributed with a mean of 5.0 cm and standard deviation of 0.4 cm. Any bolt whose diameter is less than 4.6 cm or more than 5.6 cm is sold at Ksh 42 otherwise the price is Ksh 64. If a customer orders 110 bolts from the factory, calculate the amount of money the customer is likely to spend.

(8 marks)